

Theorem:— The set of the type  $F_\sigma$  and  $G_\delta$  are measurable sets.

Proof:— Let  $E$  be a set of type  $F_\sigma$ . Then  $E$  is expressible as

$$E = \bigcup_{K=1}^{\infty} F_K$$

where  $F_K$  is a closed set.

$F_K$  is a closed set  $\Rightarrow F_K$  is a measurable set

Also an enumerable union of measurable sets is measurable.

Hence  $\bigcup_{K=1}^{\infty} F_K$  i.e.  $E$  is measurable

(ii) Let  $E$  be a set of the type  $G_\delta$  so that  $E$  can be expressed as

$$E = \bigcap_{K=1}^{\infty} G_K$$

$G_K$  is a open set  $\Rightarrow G_K$  is a measurable set

Also an enumerable intersection of measurable sets is measurable.

Hence  $\bigcap_{K=1}^{\infty} G_K$  i.e.  $E$  is measurable

Definition: Set of the Type  $F_\sigma$ :— A set  $E$  is said to be of the type  $F_\sigma$  if it is expressible as a union of an enumerable number of closed sets

$$F_K \quad \text{i.e.} \quad E = \bigcup_{K=1}^{\infty} F_K$$

Set of the Type  $G_\delta$ :— A set  $E$  is said to be of the type  $G_\delta$  if it is expressible as an intersection of an enumerable number of open sets

$$G_K \quad \text{i.e.} \quad E = \bigcap_{K=1}^{\infty} G_K$$

Evidently complement of  $F_\sigma$  is  $G_\delta$  and conversely

Borel Set:— A set  $E$  is called a Borel set if it can be obtained from closed and open sets by using a finite or an enumerable number of union and intersection operations.

Example: The sets of type  $F_\sigma$  and  $G_\delta$  are Borel sets.